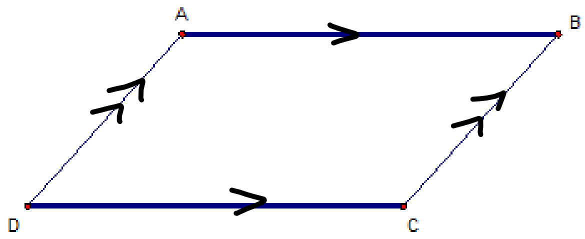
Quadrilaterals: polygon having 4 sides.

no $\parallel$ sides	1 pair of $\parallel$ sides	2 pairs of $\parallel$ sides
kite	trapezoid ↓ isosceles - the non-$\parallel$ sides are $\cong$	parallelograms rhombus has 4 $\cong$ sides rectangle has 4 right $\angle$s (4 $\cong$ $\angle$s) square has 4 $\cong$ sides and 4 right angles.

Definition to Know:

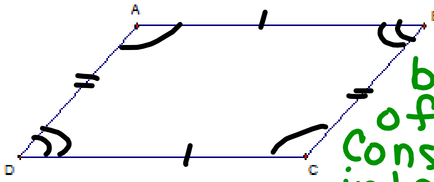
- **A *parallelogram* is a quadrilateral in which two pairs of opposite sides are parallel.**



ABCD is a parallelogram.
$\overline{AB} \parallel \overline{DC}$ and $\overline{AD} \parallel \overline{BC}$.

Theorems to Know:

- **Opposite sides of a parallelogram are congruent.**
- **Opposite angles of a parallelogram are congruent.**
- **Two consecutive angles of a parallelogram are supplementary.**
- **If a parallelogram has one right angle, then it has four right angles.**



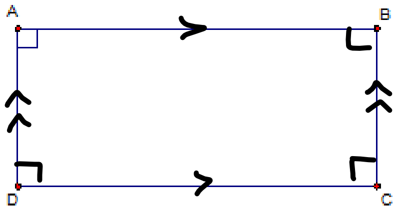
If ABCD is a parallelogram, then each of the following statements are TRUE:

$\overline{AB} \cong \overline{DC}$ and $\overline{AD} \cong \overline{BC}$

$\angle A \cong \angle C$ and $\angle B \cong \angle D$

because of the consecutive interior angles Thm.

$$\left.\begin{aligned} m\angle A + m\angle B &= 180^\circ \\ m\angle B + m\angle C &= 180^\circ \\ m\angle C + m\angle D &= 180^\circ \\ m\angle D + m\angle A &= 180^\circ \end{aligned}\right\}$$

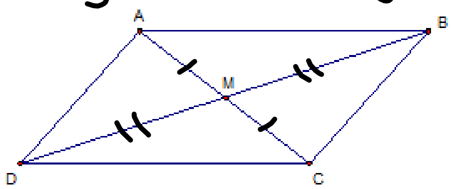


If $\angle A$ is a right angle, then $\angle B$, $\angle C$, and $\angle D$ are also right angles.

* because of the consecutive interior angles theorem.

- **The diagonals of a parallelogram bisect each other.**

Diagonals: segments that connect opposite ∠s.



If ABCD is a parallelogram, then each of the following statements are TRUE:

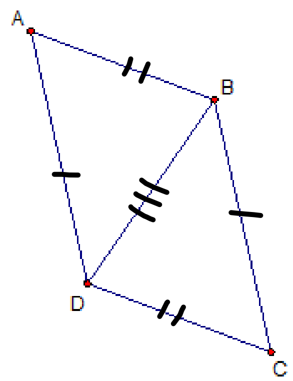
M is the midpoint of $\overline{AC}$ and $\overline{BD}$

$\overline{AM} \cong \overline{CM}$

$\overline{BM} \cong \overline{DM}$

$\overline{AC}$ and $\overline{BD}$ are the diagonals.

- **Each diagonal separates the parallelogram into two congruent triangles.**



$\overline{AD} \cong \overline{BC}$
$\overline{AB} \cong \overline{DC}$ } opposite sides of a parallelogram are $\cong$.

$\overline{BD} \cong \overline{BD}$ by Reflexive Property.

If ABCD is a parallelogram, then $\Delta ABD \cong \Delta CDB$. by SSS.

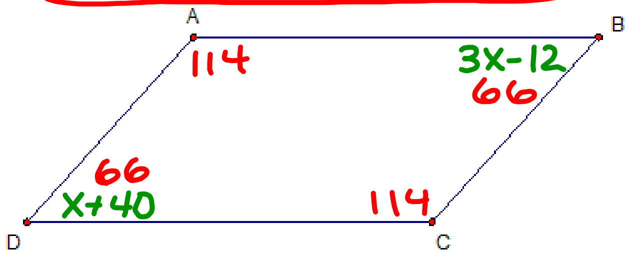
diagonal $\overline{BD}$ is drawn.

Example Problems:

1. In parallelogram ABCD, m∠ABC = 3x – 12 and m∠CDA = x + 40. Find m∠ABC, m∠CDA, m∠BCD, and m∠DAB.

m∠B m∠D

m∠B, m∠D, m∠C, m∠A



∠B ≅ ∠D
opposite angles are ≅.

$$3x-12=\cancel{1x}+40$$
$$-1x \quad -\cancel{1x}$$
$$2x-\cancel{12}=40$$
$$+\cancel{12} \quad +12$$
$$\frac{\cancel{2}x}{\cancel{2}}=\frac{52}{2}$$
$$x=26$$

m∠B = 3(26) − 12 = 66
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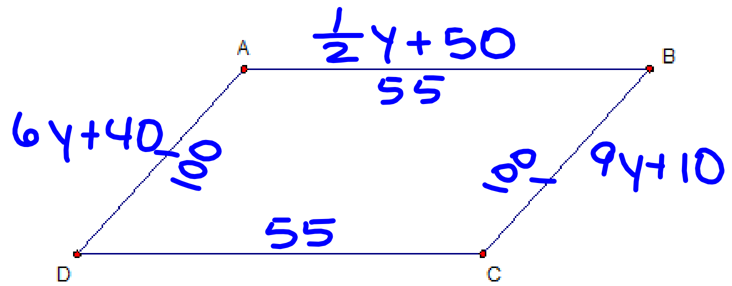
m∠D = 26 + 40 = 66

m∠A = 180 − 66 = 114

m∠C = 114

2. In parallelogram ABCD, BC = 9y + 10, AD = 6y + 40, and AB = ½y + 50. Find BC, AD, AB, and DC.



$\overline{AD} \cong \overline{BC}$

opposite sides $\cong$

$6y+40=9y+\cancel{10}$

$-10 \quad -\cancel{10}$

$\cancel{6y}+30=9y$

$-\cancel{6y} \quad -6y$

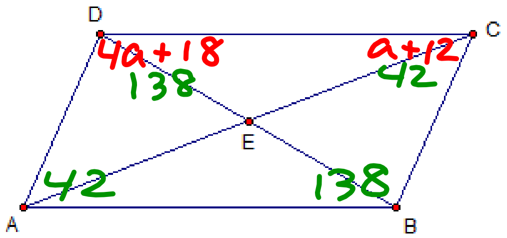
$\frac{30}{3}=\frac{\cancel{3}y}{\cancel{3}}$ $y=10$

$AD=\underbrace{6(10)}_{60}+40=100$

$BC=\underbrace{9(10)}_{90}+10=100$

$AB=\underbrace{\frac{1}{2}(10)}_{5}+50=55$

$DC=55$



Problems 3 and 4 refer to the diagram below.

3. If m∠DCB = a + 12 and m∠CDA = 4a + 18, find the degree measures of the angles of the parallelogram.

Sum of consecutive angles = 180

$m\angle DCB + m\angle CDA = 180$

$a + 12 + 4a + 18 = 180$

$5a + 30 = 180$

$-30 \quad -30$

$\frac{5a}{5} = \frac{150}{5}$

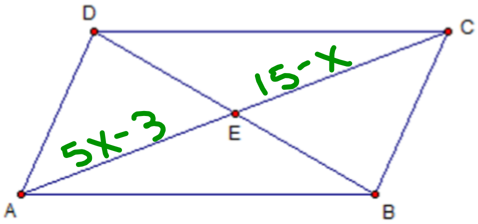
$a = 30$

$m\angle DCB = 30 + 12 = 42$

$m\angle CDA = 4(30) + 18 = 138$

120

4. If AE = 5x – 3 and EC = 15 – x, find AC.



Diagonals bisect each other.

E is the midpoint of $\overline{AC}$. $\longrightarrow$ $\overline{AE} \cong \overline{EC}$

$AE = EC$

$5x - 3 = 15 - x$

$+1x \quad\quad +x$

$6x - 3 = 15$

$+3 \quad +3$

$\frac{6x}{6} = \frac{18}{6}$ $\quad x = 3$

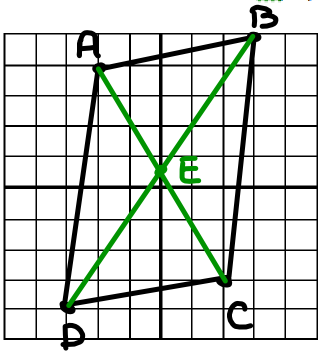
$AC = AE + EC$

$AE = 5(3) - 3 = 15 - 3 = 12$

$EC = 15 - 3 = 12$

$AC = 12 + 12 = 24$

5. Determine the coordinates of the intersection of the diagonals of Parallelogram ABCD with vertices A(-2, 4), B(3, 5), C(2, -3), and D(-3, -4).



$$\text{midpoint} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

$$\text{mp. of } AC = \left(\frac{-2+2}{2}, \frac{4-3}{2}\right) = \left(\frac{0}{2}, \frac{1}{2}\right) = (0, 0.5)$$

$$\text{mp of } BD = \left(\frac{3-3}{2}, \frac{5-4}{2}\right) = \left(\frac{0}{2}, \frac{1}{2}\right) = (0, 0.5)$$

The point of intersection, E, is (0,0.5)